277. Proposed by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

It is tacitly assumed in elementary geometry that as the number of sides of a regular polygon inscribed in a circle is increased, *in any manner*, that its perimeter has a fixed limit. Beginning with a square and then continually doubling the number of sides we get for the perimeter $2^{n+2}\sqrt{[2-E^n(0)]}$, where $E(x)\equiv\sqrt{(2+x)}$. Beginning with a hexagon we get $2^{m+1}3\sqrt{[2-E^m(1)]}$. The definition of the length of a circle assumes that these expressions have the same limit as $n\doteq\infty$ and $m\doteq\infty$. Prove it.

I. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Putting $\cos x=b$, we have $\sin\frac{1}{2}x=\sqrt{[\frac{1}{2}(1-b)]}=\frac{1}{4}\sqrt{[2-2b]}$, $\cos\frac{1}{2}x=\frac{1}{2}\sqrt{[2+2b]}$, $\sin\frac{1}{4}x=\frac{1}{2}\sqrt{[2-\sqrt{(2+2b)}]}$, $\cos\frac{1}{4}x=\frac{1}{2}\sqrt{[2+\sqrt{(2+2b)}]}$, $\sin\frac{1}{8}x=\frac{1}{2}\sqrt{\{2-\sqrt{[2+\sqrt{(2+2b)}]}\}}$, etc. Let the radius of a circle=1, then we may represent the perimeter of a regular polygon by $2^{n+2}\sin\frac{1}{2^n}x$. For $n=\infty$, this assumes the indeterminate form $\infty\doteq 0$; but we may reduce it to $\frac{2^2\sin(1/2^n)x}{1/2^n}$. Differentiating numerator and denominator with reference to n, we get $2^2x\cos(1/2^n)x$. For $n=\infty$, this becomes 2^2x. For $x=90°$, $2^2\times90=360°$.

In regard to the hexagon, we have similarly, $\frac{3\sin(1/2^n)\times120°}{1/2^n}$, which after differentiation as above, becomes $3\times120°=360°$, so that the limit in both cases is 360°.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

$E^n(0)=\sqrt{\{2+\sqrt{[2+\sqrt{(2+\sqrt{(2+,\text{ etc.}))}]}\}}$;

$E^m(1)=\sqrt{\{2+\sqrt{[2+\sqrt{(2+\sqrt{(2+,\text{ etc.}))}]}\}}$, the last term in the root being $\sqrt{3}$.

Let $E^n(0)=E^m(1)=x$. $\therefore\sqrt{(2+x)}=x$, or $x=2$.

$$\frac{2^{n+2}\sqrt{[2-E^n(0)]}}{2^{m+1}3\sqrt{[2-E^m(1)]}}=\frac{2^{n+2}\sqrt{[2-x]}}{2^{m+1}\sqrt{[18-9x]}}=\frac{2^{n+1}\sqrt{[8-4x]}}{2^{m+1}\sqrt{18-9x]}}.$$

$$\frac{2^{n+1}}{2^{m+1}}\cdot\frac{\sqrt{[8-4x]}}{\sqrt{[18-9x]}}=\frac{\sqrt{[8-4x]}}{\sqrt{[18-9x]}},\text{ when } n=m=\infty.$$

If $\frac{\sqrt{[8-4x]}}{\sqrt{[18-9x]}}=1$, $8-4x=18-9x$, or $x=2$,

the same value as found above for x. Hence the expressions are equal when n and m are indefinitely increased.

279. Proposed by C. C. WENTWORTH, C. E., Roanoke, Va.

To construct geometrically the maximum equilateral triangle circumscribed about a given triangle.